Information Horizons: AMERICAN Journal of Library And Information Science Innovation
Volume 3, Issue 11, 2025 ISSN (E): 2993-2777

Solving Invertible Equations and Higher-Degree Equations

Mamaraimov Bekzod Kadirovich, Makhmudov Azam Kudratovich, Musurmonov Maruf Akrom oglu
Teacher of mathematics at Terdu Academic Lyceum

Abstract. *Solving inverse equations and higher-order equations is one of the important areas of mathematical analysis and algebra. An inverse equation is an equation that is related to a power or function of a variable and requires finding the value of the variable. Higher-order equations are represented by polynomials of degree two or higher. This topic analyzes methods for solving equations, finding roots using discriminants and formulas, factoring, and graphical approaches. Also, methods for solving higher-order equations using complex numbers and trigonometric functions are considered. Along with theoretical foundations, problem-solving strategies and practical applications serve to strengthen mathematical knowledge. The topic is widely used in the fields of scientific research and applied mathematics.*

Keywords: *higher-order equation, polynomial, root, discriminant, factoring, complex numbers, trigonometric solution, solution methods, mathematical analysis.*

Recurrent equations (recurrent equations) and higher-order algebraic equations are one of the fundamental areas of mathematics, they are the main tool for modeling many scientific, technical, economic and natural processes. Recurrent equations describe time-varying sequences, while higher-order algebraic equations create the mathematical basis of multivariable processes, physical models, complex geometric shapes and economic situations.

This article provides a detailed scientific analysis of the types of recurrent equations, methods for solving them, the theoretical foundations of higher-order equations, the existence and properties of solutions, and numerical solutions.

A recurrent equation is a mathematical relationship in which the nth term of the sequence is determined by one or more of the terms preceding it. Its general form is:

$a_n = f(a_{n-1}, a_{n-2}, ..., a_{n-k})$

Here f is the functional connectivity, k is the regression depth.

Regression equations are used in many areas: algorithm analysis, economic growth modeling, population dynamics, signal processing, cryptography, etc.[2]

A linear regression equation has the following form:

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + ... + c_k a_{n-k}$

Example: Fibonacci sequence:

$a_n = a_{n-1} + a_{n-2}$

This equation is a linear homogeneous regression equation of degree 2.

In nonlinear equations, the function is not linear:

$a_n = a_{n-1}^2 + 3a_{n-2}$

These types of equations model complex dynamical systems.

Homogeneous equation:

$a_n — c_1 a_{n-1} — ... — c_k a_{n-k} = 0$

Inhomogeneous equation:

$a_n — c_1 a_{n-1} — ... — c_k a_{n-k} = g(n)$

Solving inverse equations is one of the important tasks in mathematics. Basic methods:

A "particular equation" is constructed for a linear homogeneous equation:

$r^k — c_1 r^{(k-1)} — ... — c_k = 0$

The appearance of the roots gives the general solution of the sequence.

In this method, the values of the sequence are calculated step by step.

Using generating functions, the sequence is reduced to analytical form.

The higher-order algebraic equation is as follows:

$a_n x^n + a_{n-1} x^{(n-1)} + ... + a_1 x + a_0 = 0$

It has n complex roots according to the fundamental algebra theorem.[3]

Properties of higher-order equations

- The number of roots is n.
- The number of real roots can be from 0 to n.
- Complex roots come in pairs.

Unsolvable equations

According to the Abel–Ruffini theorems, a general equation of degree 5 cannot be solved in radical form.

Factorization method

For example:

$x^4 - 5x^2 + 4 = 0 \rightarrow (x^2 - 1)(x^2 - 4) = 0$

Substitution method

For example: $x^4 + 3x^2 - 10 = 0$

$t = x^2 \rightarrow t^2 + 3t - 10 = 0$

Trigonometric solutions

Cubic equation:

$x^3 = px + q$

Solved by Cardano formulas.[4]

Numerical methods

Equations that do not have an analytical solution are solved using numerical methods:

- Newton's method
- Secant method
- Tangent method

In conclusion, Recurrent equations and higher-order equations form the main part of mathematical analysis. Studying their theoretical foundations allows for mathematical modeling, analysis, and forecasting of complex processes. While recurrent equations describe changes in time, higher-order equations reveal the essence of multifactorial processes. This topic is one of the fundamental branches of mathematics and plays an incomparable role in the development of science and technology.

References

1. K. I. Belousov. Algebra va matematik analiz asoslari. – Toshkent, 2018. – 320 b.

2. D. J. Lay. Linear Algebra and Its Applications. – Pearson, 2016. – 550 p.

3. Larson R., Hostetler R. Precalculus. – Cengage Learning, 2017. – 1180 p.

4. Cormen T., Leiserson C. Algorithms. – MIT Press, 2020. – 1312 p.

5. Stewart J. Calculus. – Cengage Learning, 2019. – 1360 p.